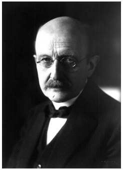
The Appearance of Events in Quantum Mechanics – a New Law of Quantum Dynamics

"The interpretation of quantum mechanics (QM) has been dealt with by many authors, and I do not want to discuss it here. I want to deal with more fundamental things" – P.A.M. Dirac

Text-book QM resulted from the profound discoveries of:



M. Planck

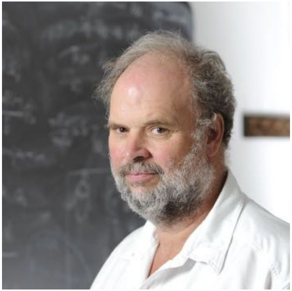
A. Einstein

W. Heisenberg

P. A. M. Dirac

Lectures 3 & 4, Venice, August 2022

Dedicated to the memory of *Vaughan Jones*, and *Detlef Dürr* – two wonderful colleagues and friends whom I miss.



V. F. R. Jones, 31.12.1952 – 6.9.2020

Detlef Dürr, 4.3.1951 – 3.1.2021

Contents and Credits

Contents:

1. What the following lectures (3 and 4) are about
2. The *ETH*-Approach to NR Quantum Mechanics (*QM*)
3. Huygens' Principle and the *Principle of Diminishing Potentialities*
4. Results for Limiting Models, as the speed of light $c \to \infty$

Summary and conclusions

Credits:

I wish to thank my collaborators and in particular my last PhD student *Baptiste Schubnel* and my friend *Alessandro Pizzo* for the joy of joint efforts on projects related to this lecture, as well as many colleagues at numerous institutions for instructive (and sometimes rather controversial) discussions on *QM*. Ideas of the late *Rudolf Haag* have proven fruitful in my work. – I am grateful to *the organizers* of this series for giving me an opportunity to present ideas and results – alas, not widely appreciated in the community, yet – on problems that are undoubtedly fundamental.

1. What the Following Lectures are About

Summary: Our purpose is to extend the standard formalism of *QM* and complete it in such a way that the resulting theory *makes sense*. The extension, yielding a *new Law of Nature*, is called

ETH - **Approach** to *QM*.

The *ETH* - Approach to *QM* supplies the fourth one of *four pillars* *QM* rests upon: (i) Physical quantities characteristic of a physical system are represented by s.-a. linear operators; (ii) the time evolution of operators representing physical quantities is given by the *Heisenberg equations*; (iii) meaningful notions of *Potential* and *Actual Events* and of *states*; and

(iv) a general statistical Law for the Time Evolution of states.

Core of talk: Besides sketching the *ETH*-Approach to *QM*, I will discuss simple models of a very heavy atom coupled to the radiation field in a limit where the speed of light tends to ∞, illustrating the *ETH*-Approach.

General goal: I am determined to remove some of the **enormous confusion** befuddling many colleagues who claim to work on the foundations of QM... Of course, hardly anybody expects that I will succeed – I do!

Topics to be Addressed – Today or in the Future

- Sketch a **novel completion** of Quantum Mechanics (*QM*), called *"ETH-Approach* to *QM*" – introducing:
- Sharp notions of: • *Isolated* physical systems in *QM* • *States* of physical systems (↗ Gleason, Maeda) • *Potential* - and *Actual Events* in *QM* (↗ Haag) – and discussing the:
- **Inadequacy of Schrödinger Eq.** → *ETH*-Approach predicts *statistical law* for time evolution of *states* of physical systems, confirming **probabilistic nature of** *QM* → *"quantum branching processes."*
- *Clarify the role of *(Space-)Time* and of the electromagnetic (and gravitational) field in *QM*.

  The *ETH* - Approach predicts that a consistent *Quantum Theory of Events* is necessarily *relativistic*, and space-time must be *even-dimensional* ...

Today's focus: **Non-Relativistic QM**; but features of *local relativistic QT* are used to motivate some constructions; in particular, our models illustrating *ETH*-Approach to *QM* are inspired by *relativistic QED* !

The discussion of these models is among the main topics of lecture.

Direct versus Indirect Measurements

Besides leading to a precise law for the *time evolution of states* in *QM*, a key purpose of the *ETH*- Approach is to solve the infamous **"measurement problem"** of *QM*; namely the problem of developing a theory of *direct/projective measurements* (highlighting the role of the e.m. field and free of internal inconsistencies). – I will not treat *th. of indirect/weak measurements*, which is well understood, *assuming* that $\exists$ good theory of *direct/projective measurements* of probes! ($\nearrow$ Kraus et al.)

Examples of indirect measnts.: Haroche-Raimond exp.; particle tracks ... (Mass-Küm, Ba-Be, BFFSch, BCrFFSch, ...; Fi-Te, BBenFF, BenFF, ...)

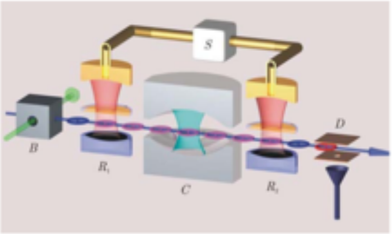
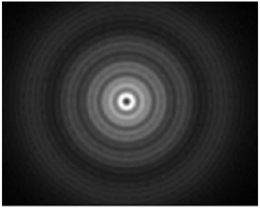


Fig. 4: Experimental setup to study microwave field states with the help of circular Rydberg atoms (see text).

The role of light in the emergence of a classical world

Apparently, an *illumination* by light of the cavity to the right of the screen containing the double slit has the effect that *"electron waves"* are converted to *"corpuscles"* by eliminating interference effects (which also illustrates the *"retarded choice paradox"* of *Wheeler*): The quantum world, which involves *potentialities* repres. by non-commuting ops. giving rise to *interference* and to Heisenberg's uncertainty relations, approaches the *factual classical world!* – This becomes strikingly manifest by considering a spherically symmetric ball of radioactive material emitting α-particles into a cavity (*Darwin, Mott, ... , Figari-Teta, (B)BFF*):



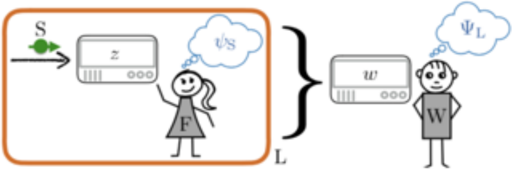
Dark cavity

Illuminated cavity

→ Must understand the special role of light in producing events, facts and "classicality"!

Recap: The Schrödinger Equation does *Not* Describe the *Time Evolution* of *States* in *QM*

Recall the *Wigner's friend paradox* ($\nearrow$ Wigner, Hardy, F-R, ...):



Courtesy *Frauchiger & Renner*

F measures the spin of the green particle in the z-direction. After a successful measurement, but *without knowing* its result, F makes predictions of future measnts. using a mixed state, while W uses *unitary evolution* of the *pure* initial state of the entire lab, including F, to make predictions. The statistics of future measurement outcomes predicted by F and W are then **contradictory**. – *Resolution of paradox:*

It is **not** true that the state of the lab evolves in time according to the (deterministic) Schrödinger equation!

$\rightarrow$ Thus, we must find the law of evolution of states in *QM*!

2. The *ETH*-Approach to NR Quantum Mechanics

Purpose: Clarify the notions of *states* in *QM*, of *Potential* & *Actual Events* featured by *isolated, open physical systems* $\rightarrow$

$$\boxed{q.m.\ \textbf{Law}\ \textit{describing the stochastic evolution of states}}$$

Let S be a physical system. **Physical quantities** characteristic of S, including *potential events* in S, are represented by certain abstract *bounded, selfadjoint linear operators*

$$\widehat{X} = \widehat{X}^* \in \mathcal{O}_S \ \text{ (a family of operators)},$$

where the only properties of $\mathcal{O}_S$ are that it contains $\mathbf{1}$ and that if $\widehat{X} \in \mathcal{O}_S$ and F is a bounded continuous function then $F(\widehat{X}) \in \mathcal{O}_S$.

Time is a fundamental quantity in NR physics. It is described by $\mathbb{R}$, or, as in the following, by $\mathbb{Z}$, and parametrizes evolution of S. Associated with every time t there is a representation of $\mathcal{O}_S$ by concrete selfadjoint operators on a separable Hilbert space $\mathcal{H}_S$:

$$\mathcal{O}_S \ni \widehat{X} \mapsto X(t) = X(t)^* \in B(\mathcal{H}_S)\,.$$

Physical Quantities and Their Time Evolution in the Heisenberg Picture

[In concrete examples of physical systems, $X(t)$ can be localized in space and in time – *Haag* talks of "local observables", *Bell* of "local beables":

$$X(t) = \int_I d\tau \int_{\mathbb{R}^3} d\mathbf{x}\, \mathfrak{x}(\tau + t, \mathbf{x})\, h(\tau, \mathbf{x})\,,$$

where $\mathfrak{x}(\tau, \mathbf{x})$ is a hermitian operator-valued distribution on $\mathcal{H}_S$ (e.g., a particle density, a component of a spin density, or an energy density, ...); $h(\tau, \mathbf{x})$ is a real test function with support in a compact interval, I, of the time axis. We then say that $X(t)$ is *localized in the time interval* $I + t$.]

$\rightarrow$ Concrete examples motivate our assumption that *every operator* X *representing a physical quantity* $\widehat{X} \in \mathcal{O}_S$ *is localized in a compact interval*, denoted I_X, *of the time axis*; ($I_{F(X)} = I_X$, for F as above).

In the *Heisenberg picture*, time evolution of operators $X(t)$ representing physical quantities $\widehat{X}$ of an *isolated* physical systems S is described by unitary conjugation with the *propagator* of the system (H: Hamiltonian):

Algebras Generated by Operators Representing Physical Quantities Localized in Compact Intervals of Time

$$X(t') = e^{i(t'-t)H/\hbar} X(t) e^{-i(t'-t)H/\hbar}, \qquad \text{for} \;\; t, t' \;\text{in}\; \mathbb{R}\,. \tag{1}$$

Let I be an arbitrary interval of *future times*, i.e., $I \subset [t_0, \infty)$, where t_0 is the *present*. We define $\mathcal{E}_I$ to be the *-algebra generated by *arbitrary finite sums of arbitrary finite products of operators*

$$\left\{ X \,\middle|\, X \text{ represents } \widehat{X} \in \mathcal{O}_S \text{ at some time } \geq t_0, \text{ with } I_X \subseteq I \right\}.$$

We define

$$\mathcal{E}_{\geq t} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I}, \quad \text{and} \quad \mathcal{E} := \overline{\bigvee_{t \in \mathbb{R}} \mathcal{E}_{\geq t}}^{\|\cdot\|}, \tag{2}$$

where the algebras $\mathcal{E}_{\geq t}, t \in \mathbb{R}$, are assumed to be *weakly* closed![1]
By definition,

$$\mathcal{E}_I \supseteq \mathcal{E}_{I'} \;\; \text{if} \;\; I \supseteq I'\,, \quad \mathcal{E}_{\geq t} \supseteq \mathcal{E}_{\geq t'} \;\; \text{if} \;\; t' > t\,.$$

[1] Passing to von Neumann algebras is convenient, because the spectral projections of any element of the algebra will then also belong to the algebra!

The Principle of Diminishing Potentialities

Definition 1: Let S be an isolated physical system. *Potential (future) Events* in S that might actualize at a time $t \geq t_0$ are special kinds of physical quantities, namely elements of *partitions of unity*, $\mathfrak{F}_t$,

$$\mathfrak{F}_t = \{\pi_\xi \,|\, \xi \in \mathfrak{X}\} \subset \mathcal{E}_{\geq t}, \quad t \geq t_0, \ \mathfrak{X} \text{ countable }, \sum_{\xi \in \mathfrak{X}} \pi_\xi = \mathbf{1},$$

by *disjoint orthogonal projections*, $\pi_\xi = \pi_\xi^*$ on $\mathcal{H}_S$, with $\pi_\xi \cdot \pi_\eta = \delta_{\xi\eta}\pi_\xi$.

An *isolated* system S is *defined* in terms of a co-filtration $\{\mathcal{E}_{\geq t} \,|\, t \in \mathbb{R}\}$ of algebras generated by *Potential Events* satisfying Eq. (1).

The *Principle of Diminishing Potentialities (PDP)* is the statement that

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'}, \quad \text{whenever } t' > t \geq t_0.} \tag{3}$$

This principle characterizes *isolated open* systems. It will be shown to hold in simple models discussed later in this talk. (*Closed* systems are ones for which $\mathcal{E}_{\geq t} \equiv \mathcal{E}$ is *independent* of t ...)

A **state** of S at time t is given by a *quantum probability measure* on the lattice of orthogonal projections in $\mathcal{E}_{\geq t}$, i.e., a functional, ω_t, with props.:

Potentialities and Actualities ($\nearrow$ Aristotle)

- ω_t assigns to every orthogonal projection $\pi \in \mathcal{E}_{\geq t}$ a non-negative number $\omega_t(\pi) \in [0,1]$, with $\omega_t(\mathbf{1}) = 1$,
- ω_t is *additive*, in the sense that

$$\sum_{\pi \in \mathfrak{F}_t} \omega_t(\pi) = 1, \quad \forall \text{ partitions of unity } \mathfrak{F}_t \subset \mathcal{E}_{\geq t}\,. \tag{4}$$

Remark: *Gleason*'s theorem (as generalized by *Maeda*) says that states, ω_t, of S at time t, as specified above, are *positive, normal, normalized linear functionals* on $\mathcal{E}_{\geq t}$, i.e., *states* on $\mathcal{E}_{\geq t}$ in the usual sense.

Note: The "initial state" of S may be *pure*; but, since $\mathcal{E}_{\geq t} \subsetneqq B(\mathcal{H}_S)$, $\forall t < \infty$, assuming that *(PDP)* holds, ω_t will generally be a *mixed* state on $\mathcal{E}_{\geq t}$: *Entanglement!* This observation opens the door towards a natural notion of *Actual Events* – "*actualities*" – in our formalism and to a theory of direct/projective measurements and observations.

In accordance with the *"Copenhagen interpretation"* of *QM*, we say that some *Potential Event* in a partition of unity $\mathfrak{F}_t = \{\pi_\xi | \xi \in \mathfrak{X}\} \subset \mathcal{E}_{\geq t}$ (see Def. 1, last slide) *actually happens* in the interval $[t, \infty)$ of times,

The Centralizer of a State and its Center

i.e., becomes an *Actual Event* setting in at time t, iff

$$\boxed{\omega_t(A) = \sum_{\xi \in \mathfrak{X}} \omega_t(\pi_\xi \, A \, \pi_\xi), \quad \forall A \in \mathcal{E}_{\geq t}\,,} \tag{5}$$

no off-diagonal elements on R.S. of (5) – *incoherent* superposition!

Next, we render the meaning of Eq. (5) precise.
Let $\mathfrak{M}$ be an algebra, and let ω be a state on $\mathfrak{M}$. We define the *centralizer* of a state ω on $\mathfrak{M}$ by

$$\mathcal{C}_\omega(\mathfrak{M}) := \left\{ X \in \mathfrak{M} \,\middle|\, \omega([A, X]) = 0, \ \forall A \in \mathfrak{M} \right\}$$

Note that $\mathcal{C}_\omega(\mathfrak{M})$ is a subalgebra of $\mathfrak{M}$ and that ω is a normalized trace on $\mathcal{C}_\omega(\mathfrak{M})$ ... ! The *center*, $\mathcal{Z}_\omega(\mathfrak{M})$, of $\mathcal{C}_\omega(\mathfrak{M})$ is defined by

$$\mathcal{Z}_\omega(\mathfrak{M}) := \left\{ X \in \mathcal{C}_\omega(\mathfrak{M}) \,\middle|\, [X, A] = 0, \, \forall A \in \mathcal{C}_\omega(\mathfrak{M}) \right\}. \tag{6}$$

$\rightarrow$ Good general notion of *Actual Events* – "actualities": Let S be an isolated physical system. In (6) we set $\mathfrak{M} := \mathcal{E}_{\geq t}$, $\omega := \omega_t$.

Actual Events and time evolution of states

Definition 2: Given that ω_t is the state of S at time t, an *Actual Event* is setting in at time t iff $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains at least two non-zero orthogonal projections, $\pi^{(1)}, \pi^{(2)}$, which are disjoint, i.e., $\pi^{(1)} \cdot \pi^{(2)} = 0$, and have non-vanishing *"Born probabilities"*, i.e.,

$$0 < \omega_t(\pi^{(i)}) < 1\,, \quad \text{for } i = 1, 2\,. \qquad \square$$

Let us suppose for simplicity that $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ is generated by a partition of unity $\mathfrak{F}_t = \{\pi_\xi | \xi \in \mathfrak{X}_{\omega_t}\}$ of orth. proj., where $\mathfrak{X}_{\omega_t} = \text{spec}[\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})]$ is a countable set. Then *Eq. (5) holds true!*

The **Law** describing the *time evolution of states* in *QM* is derived from the following *State Reduction-, or Collapse Postulate*, which makes precise mathematical sense if *time* is **discrete** (!):
Let ω_t be the state of S on $\mathcal{E}_{\geq t}$. Let dt denote a time step; (dt may be positive if time is discrete; otherwise we will let dt tend to 0 at the end of our constructions).

The State-Reduction (Collapse) Postulate

We define a state on the algebra $\mathcal{E}_{\geq t+dt}$ by setting

$$\overline{\omega}_{t+dt} := \omega_t\big|_{\mathcal{E}_{\geq t+dt}} .$$

Axiom CP: *Let $\mathfrak{F}_{t+dt} := \left\{\pi_\xi \,|\, \xi \in \mathfrak{X}_{\overline{\omega}_{t+dt}}\right\}$ be the partition of unity generating the spectrum, $\mathfrak{X}_{\overline{\omega}_{t+dt}}$, of $\mathcal{Z}_{\overline{\omega}_{t+dt}}(\mathcal{E}_{\geq t+dt})$.*

Then **'Nature'** *replaces the state $\overline{\omega}_{t+dt}$ on $\mathcal{E}_{\geq t+dt}$ by a state*

$$\omega_{t+dt}(\cdot) \equiv \omega_{t+dt,\xi}(\cdot) := \overline{\omega}_{t+dt}(\pi_\xi)^{-1} \cdot \overline{\omega}_{t+dt}(\pi_\xi(\cdot)\pi_\xi)\,,$$

for some $\xi \in \mathfrak{X}_{\overline{\omega}_{t+dt}}$, with $\overline{\omega}_{t+dt}(\pi_\xi) \neq 0$.

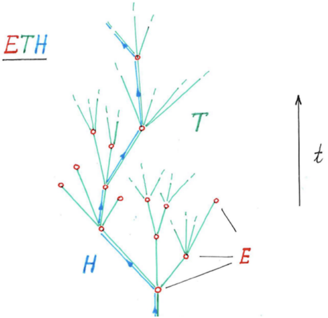
The probability, $prob_{t+dt}(\xi)$, for the state $\omega_{t+dt,\xi}$ to be selected by Nature as the state of S at time $t+dt$ is given by

$$prob_{t+dt}(\xi) = \overline{\omega}_{t+dt}(\pi_\xi) \qquad (Born's\ Rule) \qquad \square \tag{7}$$

Remark: The mathematical theory obtained when the time step, dt, tends to 0 is not analyzed rigorously, yet. $\rightarrow$ Challenge for math.!

A Metaphoric Picture of the Time Evolution of States in QM, as Predicted by the “ETH-Approach”

Apparently, the time-evolution of *states* of a phys. system is described by a *stochastic branching process*, with branching rules det. by **Axiom CP**. This can be made precise, mathematically, if time is discrete, and it leads to a good notion of *projective measurements*; see models discussed later.



Rudolf Haag

E: “Events”, T: “Trees” of possible states, H: “Histories” of states.

This is different from and supercedes the “Decoherence- and the Many-Worlds mumbo-jumbo”!

Events and their detection in measurements

We have characterised an *isolated open system* S in terms of a filtration of algebras

$$\{\mathcal{E}_{\geq t}\}_{t\in\mathbb{R}} ,$$

with

$$\boxed{\mathcal{E}_{\geq t} \underset{\neq}{\supset} \mathcal{E}_{\geq t'} , \qquad \text{whenever} \;\; t' > t} \tag{8}$$

The flow of time in S, (i.e., the time evolution of S in the Heisenberg picture) is encoded in the *proper* embeddings (8), which, in an *autonomous* system S, are completely determined by its *Hamiltonian*.

However, the characterisation of S given in (8) is incomplete! To retrieve physical information from (8) and from our definition of *events*, we must specify operators that represent *"physical quantities"* characteristic of S and – when observed/measured – may *signal the occurrence of events*. Let

$$\mathcal{O}_S := \{\hat{X}_\iota | \iota \in \mathcal{I}_S\} \tag{9}$$

be a list/set of abstract linear operators representing physical quantities characteristic of S; (usually, $\mathcal{O}_S$ is not a linear space, let alone an alg.).

Measurements of physical quantities

For any operator $\hat{Y} \in \mathcal{O}_S$ and any time t, there is a concrete self-adjoint operator $Y(t) \in \mathcal{E}_{\geq t}$ representing $\hat{Y}$ at time t; (i.e., $\exists$ a repr. of $\mathcal{O}_S$ by operators on $\mathcal{H}_S$, $\forall t \in \mathbb{R}$). For an *autonomous* system S, the operators $Y(t)$ and $Y(t')$ are conjugated to one another by the *propagator* of S.

Suppose that, at some time t, an *event* happens; i.e., $\exists$ a partition of unity, $\{\pi_\xi | \xi \in \mathcal{X}_{\omega_t}\} \subseteq \mathcal{Z}_{\omega_t} \subset \mathcal{E}_{\geq t}$, by disjoint (commuting) orthogonal projections, as above, containing ≥ 2 elements with strictly positive Born probabilities representing *possible events* (one of which *actually happens*). Let $\hat{Y} \in \mathcal{O}_S$, and let $Y(t) = \sum_{\eta \in spec(\hat{X})} \eta \, \Pi_\eta(t)$ (spectral dec. of Y(t)) be the operator representing $\hat{Y}$ at time t. If the "distance"[2]

$$\mathrm{dist}\Big(\Pi_\eta(t), \langle \pi_\xi | \xi \in \mathcal{X}_{\rho_t} \rangle\Big) \text{ is "very small"}, \ \forall \eta \in spec(\hat{Y}), \tag{10}$$

then we say that the *physical quantity* $\hat{Y} \in \mathcal{O}_S$ is *observed/measured* after time t, because the state of S just after time t is then an approximate eigenstate of $Y(t)$. The measurement of $\hat{Y}$ is a *signal of an event happening* at time t. ...

[2] defined, e.g., in terms of conditional expectations

Remarks on the *ETH*-Approach

1. *Actual Events* might be recorded by *"projective measurements"* of *physical quantities* $\widehat{Y} \in \mathcal{O}_S$, as just sketched.

2. A *passive state*, ω_t, is a state for which $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ consists of only two projections, π and $\pi^\perp$, with $\omega_t(\pi) = 1$ and $\omega_t(\pi^\perp) = 0$. If ω_t is passive it does not feature any event at time t. If ω_t is time- transl. invariant & passive then $\omega_{t'}$ is passive, $\forall t' > t$. *Thermal equilibrium states* and states of *closed systems* are passive at all times.

3. A microscopic system only **weakly** coupled to fields with ∞ many degrees of freedom (such as the e. m. field) has the property that, for most times t, $\mathcal{Z}_{\omega_t}(\mathcal{E}_{\geq t})$ contains a projection, π_0, with the property that $\omega_t(\pi_0) \simeq 1$, while $\omega_t(\pi) \simeq 0, \forall \pi \perp \pi_0$ in $\mathcal{E}_{\geq t}$. The state of such a system is then nearly *constant* in time, in the Heisenberg picture (i.e., evolves approximately according to the *Schrödinger eq.* in the Schrödinger picture), *except at rare instances* when an unlikely event makes it jump. For purely *entropic reasons*, such rare jumps must occur at a *non-zero* rate, unless the state of the system is a time-translation invariant passive state.

3. Huygens' Principle and *PDP*

This discussion is inspired by *ETH*-Approach to relativistic *QT* possibly sketched later.

Huygens' Principle for massless modes (photons, gravitons, ...) in isolated physical systems

$\Rightarrow$ *Principle of Diminishing Potentialities!*

S : Isolated system consisting, for example, of a static atom located at $\mathbf{x} = 0$, coupled to the *electromagnetic field.* Concretely:

- Atom has M internal energy levels, Hilbert space $\mathfrak{h}_A \simeq \mathbb{C}^M$.
- Hilbert space of e.m. field is *Fock space*, $\mathfrak{F}$, of photons; e.m. field described by field tensor, $F_{\mu\nu}(\tau, \mathbf{x})$, with property that, for real-valued test functions $\{h^{\mu\nu}\}$ on space-time, the operator

$$F(h) := \int_{\mathbb{R}\times\mathbb{R}^3} d\tau\, d\mathbf{x}\, F_{\mu\nu}(\tau, \mathbf{x})\, h^{\mu\nu}(\tau, \mathbf{x})$$

is self-adjoint on $\mathfrak{F}$ and satisfies locality. The usual Hamiltonian of the free e.m. field is denoted by H_0; with $H_0 = H_0^* \geq 0$ on $\mathfrak{F}$. Hilbert space of S:

$$\mathcal{H}_S := \mathfrak{F} \otimes \mathfrak{h}_A\,.$$

A Concrete Model

Let $V_t^{\pm}$ be the forward/backward light cone above the space-time point $(t, \mathbf{x} = 0)$. We define

Space-time diamonds: $D_{[t,t']} := V_t^+ \cap V_{t'}^-$, with $t' > t$.

Bounded functions of field operators $F(h)$, $\mathrm{supp}(h^{\mu\nu}) \subseteq D_{[t,t']}$, generate a (von Neumann) algebra $\mathcal{A}_{I=[t,t']}$. We then define

$$\mathcal{D}_I^{(0)} := \mathcal{A}_I \otimes \mathbf{1}\big|_{\mathfrak{h}_A}, \qquad \mathcal{E}_I^{(0)} := \mathcal{A}_I \otimes B(\mathfrak{h}_A),$$
$$\mathcal{E}_{\geq t}^{(0)} := \overline{\bigvee_{I \subset [t,\infty)} \mathcal{E}_I^{(0)}}. \tag{8}$$

PDP holds for non-interacting system: Setting $I := [t, t']$, one has that

$$\boxed{\left[\mathcal{E}_{\geq t'}^{(0)}\right]' \cap \mathcal{E}_{\geq t}^{(0)} = \mathcal{D}_I^{(0)} \quad (\text{an } \infty - \text{dim. algebra!})} \tag{9}$$

Remark: Follows from *Huygens' Principle*, namely from the fact that

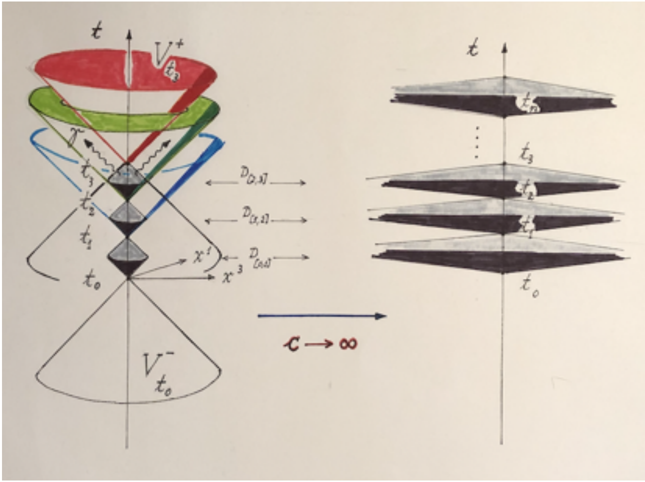
$$[F_{\mu\nu}(x), F_{\rho\sigma}(y)] = 0, \quad \text{unless } x - y \text{ is light-like.}$$

From now on, we discretize time: $t_n := n \in \mathbb{Z}$. Speed of light: c.

Interacting Propagator, Γ / Illustration of *HP*

To describe interactions, pick a unitary operator $U \in \mathcal{E}^{(0)}_{[0,1]}$, and define $\Gamma := e^{-iH_0}U$. Then the propagator of the coupled system is given by

$$\Gamma^n = e^{-inH_0}U(n), \ (\Gamma^n)^* = \Gamma^{-n}, \ U(n) = \cdots, \ n = 0, 1, 2, \ldots \qquad (10)$$



PDP for the Interacting Model

It suffices to consider time evolution for times $t \geq t_0 := 0$. We define

$$\mathcal{E} := \mathcal{E}^{(0)}_{\geq 0}, \quad \mathcal{E}_{\geq n} := \left\{ \Gamma^{-n} X \Gamma^{n} \,\middle|\, X \in \mathcal{E} \right\}. \tag{11}$$

It is straightforward to verify *PDP* for the *interacting model*: Using (10) and (11), one shows that

$$\left[\mathcal{E}_{\geq n'}\right]' \cap \mathcal{E}_{\geq n} \simeq \mathcal{D}_{[n,n']}, \quad \text{for} \quad n' > n, \tag{12}$$

where $\mathcal{D}_{[n,n']} := \left\{ U(n')^* X \, U(n') \,\middle|\, X \in \mathcal{D}^{(0)}_{[n,n']} \right\}$. Note that

$$\boxed{\Gamma^{n-n'} \mathcal{E}_{\geq n} \Gamma^{n'-n} = \mathcal{E}_{\geq n'} \subsetneqq \mathcal{E}_{\geq n}, \quad \text{for} \quad n' > n.}$$

Preparing the system in an initial state, ω_0, at time $n = 0$, we would like to determine the *stochastic time evolution* of states, ω_t, predicted by the law encoded in *Definition 2* (Actual Events) and **Axiom CP** of Sect. 2.

Stochastic Time Evolution with Memory

We consider an *Example:*

$$\omega_0(X) := \mathrm{tr}_{\mathcal{H}_S}\big(\big[|0\rangle\langle 0| \otimes \Omega\big]\, X\big), \quad X \in \mathcal{E}\,,$$

with $|0\rangle$ the *vacuum vector* in $\mathfrak{F}$, and Ω a density matrix on $\mathfrak{h}_A$.

The state ω_0 does *not* entangle the atom with the e.m. field. Yet, interactions *will* entangle them in the course of time, as expected. Since the vacuum $|0\rangle\langle 0|$ is not a *"product state," stochastic time evolution* of states of S exhibits *memory effects* – explicit control rather difficult!

Matters simplify drastically in the limit where the speed of light, c, tends to ∞, which we consider next.

In the limit $c \to \infty$, the regions $D_{[k,k+1]}$ approach **time slices**, $k \leq t \leq k+1$ (Fig.!), and the algebras $\mathcal{D}^{(0)}_{[k,k+1]}$ are given by

$$\mathcal{D}^{(0)}_{[k,k+1]} \simeq B(\mathcal{H}_k), \;\; \mathcal{H}_k \overset{e.g.}{=} \mathbb{C}^N, \text{ for some } N \leq \infty\,, \forall\, k. \qquad (13)$$

4. Results for Limiting Models, as $c \to \infty$

with *Alessandro Pizzo*

As $c \to \infty$: electromagnetic field $\to$ "*R*-field" ($N < \infty$, henceforth).

Follow evolution of S only for $t \geq 0$. Pick an orthonormal basis $\{\phi_j\}_{j=0}^{N-1}$ in $\mathbb{C}^N$; $\mathcal{S}_{fin} :=$ set of sequences $\underline{k} := \{k_n\}_{n=0}^{\infty}$, with $k_n = 0, \ldots, N-1$ and $k_n = 0$, except for *finitely many* values of $n \in \mathbb{Z}_+$. For $\underline{k} \in \mathcal{S}_{fin}$, we define

$$\Phi_{\underline{k}} := \bigotimes_{n=0}^{\infty} \phi_{k_n}\,, \qquad \Phi_{\underline{0}} : \text{"}\textit{vacuum}\text{" (reference vector)}. \tag{14}$$

The Hilbert space, $\mathfrak{F}_{\underline{0}}$, of the R-field is then given by the closure of the space of finite linear combinations of vectors $\{\Phi_{\underline{k}} \,|\, \underline{k} \in \mathcal{S}_{fin}\}$ in the norm determined by the scalar product defined by

$$\langle \Phi_{\underline{k}}, \Phi_{\underline{k'}} \rangle := \prod_{n=0}^{\infty} \delta_{k_n, k'_n}\,. \tag{15}$$

We then set

$$\mathcal{H}_S := \mathfrak{F}_{\underline{0}} \otimes \mathfrak{h}_A\,.$$

The Propagator of the Model

We define a *shift*, σ, on $\mathcal{S}_{fin}$ by $\sigma(\underline{k})_n := k_{n+1}$, and define the *time-1 propagator of the R-field* (before coupling to A) by

$$\mathfrak{S}\Phi_{\underline{k}} := \Phi_{\sigma(\underline{k})}, \quad \underline{k} \in \mathcal{S}_{fin}\,,$$

extended to $\mathfrak{F}_{\underline{0}}$ by linearity. Note that $\mathfrak{S}\Phi_{\underline{0}} = \Phi_{\underline{0}}$. The *time-1 propagator of the atom* (before couplng to R-field) is given by a unitary operator V on $\mathfrak{h}_A$, and we set $\Gamma_0 := \mathfrak{S} \otimes V$.

To introduce *interactions*, pick unitary U on $\mathbb{C}^N \otimes \mathfrak{h}_A$ and define

$$\begin{aligned} U_1 := U\big|_{\mathcal{H}_0}, U_k := \Gamma_0^{1-k} U_1 \Gamma_0^{k-1},\ k = 1, 2, \dots, \\ U(n) := U_n \cdots U_1,\ n = 1, 2, \dots, \end{aligned} \tag{16}$$

Interacting propagator of model given by $\left\{\Gamma^n\right\}_{n=0,1,2,\dots}$, where

$$\Gamma := \Gamma_0 U_1 \ \text{(unitary)} \ \Rightarrow\ \Gamma^n = \Gamma_0^n U(n),\ \forall\, n \in \mathbb{Z}_+\,, \tag{17}$$

Time Evolution of States, According to "*ETH*"

Algebras:

$$\mathcal{E} := \overline{\left\{\text{finite sums of ops. } F \otimes C \,\middle|\, F \in B(\mathfrak{F}_{\underline{0}}), C \in B(\mathfrak{h}_A)\right\}},$$
$$\mathcal{E}_{\geq n} := \left\{\Gamma^{-n} X \,\Gamma^{n} \,\middle|\, X \in \mathcal{E}\right\}, \quad n = 0, 1, 2, \ldots \tag{18}$$

Initial state: Let Ω_0 be a density matrix on $\mathfrak{h}_A$. We set

$$\omega_0(X) := \langle \Phi_{\underline{k}}, F\, \Phi_{\underline{k}} \rangle \cdot \mathrm{tr}\big(\Omega_0 \cdot C\big), \text{ with } \underline{k} \in \mathcal{S}_{fin}, X = F \otimes C \in \mathcal{E}.$$

Our aim is to determine the time evolution of ω_0 according to the Law ($\nearrow$ *Definition 2* & **Axiom CP**) of the *ETH*-Approach. Using *induction in time* n, we find that state, ω_n, on $\mathcal{E}_{\geq n}$ is given by

$$\omega_n\big(\Gamma^{-n} X \,\Gamma^{n}\big) = \langle \Phi_{\sigma^n(\underline{k})}, F\, \Phi_{\sigma^n(\underline{k})} \rangle \cdot \mathrm{tr}\big(\Omega_n \cdot C\big), \tag{19}$$

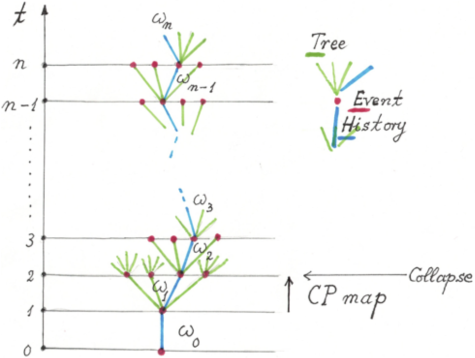
where Ω_n is a density matrix on $\mathfrak{h}_A \propto$ an orthogonal projection; $\big\{\Omega_n\big\}_{n=0,1,2,\ldots} \sim$ sample path of a *stochastic branching process*:

Time Evolution of States – Summary

The *stochastic time evolution* of states in our model,

$$\omega_0 \to \cdots \to \omega_{n-1} \to \omega_n \to \cdots, \qquad \omega_0 \text{ as above,}$$

is described in terms of a *quantum Markov chain* which depends on $\underline{k} \in \mathcal{S}_{fin}$ and acts on density matrices of atom. The sample paths, $\{\Omega_n\}_{n=0}^{\infty}$, are obtained by *"unravelling"* this Markov chain; (next slide).



A Very Simple Explicit Model

A simple example of an operator U *describing interactions "$A - R$":*

Let $\{Q_m\}_{m=1}^{M}$ be a partition of unity by orthogonal projections on $\mathfrak{h}_A$ – for ultimate simplicity, $Q_m = |\psi_m\rangle\langle\psi_m|$, where $\{\psi_m\}_{m=1}^{M}$ is a CONS. Let $T^{(m)}$ be a unitary operator on $\mathbb{C}^N$, $\forall\, m = 1, \dots, M$. We define

$$U := \sum_{m=1}^{M} T^{(m)} \otimes Q_m \,.$$

We follow *stochastic evolution* of initial state ω_0 according to *ETH*. It turns out that if ω_0 is chosen as above then, after $n = 1, 2, \dots$ time steps, the formula for the state ω_n, applied to operators of the form $\Gamma^{-n}(F \otimes C)\Gamma^n \in \mathcal{E}_{\geq n}$, is given by

$$\boxed{(\mathcal{I}_n) \qquad \omega_n\big(\Gamma^{-n}(F \otimes C)\Gamma^n\big) = \langle \Phi_{\sigma^n(\underline{k})}, F\,\Phi_{\sigma^n(\underline{k})}\rangle \cdot \mathrm{tr}\big(\Omega_n \cdot C\big)} \qquad (20)$$

where $\Omega_n \propto$ orthogonal projection.

We now explain the *induction step* $(\mathcal{I}_n) \Rightarrow (\mathcal{I}_{n+1})$. We first consider the restriction of ω_n to the algebra $\mathcal{E}_{\geq(n+1)}$:

$$\omega_n\big(\underbrace{\Gamma^{-(n+1)}(F \otimes C)\Gamma^{n+1}}_{\equiv X \in \mathcal{E}_{\geq(n+1)}}\big) = \langle \Phi_{\sigma^{n+1}(\underline{k})}, F\,\Phi_{\sigma^{n+1}(\underline{k})}\rangle \cdot \mathrm{tr}\big(\widehat{\Omega}_{n+1} \cdot C\big),$$

The Induction Step

where the density matrix $\widehat{\Omega}_{n+1}$ is given by

$$\widehat{\Omega}_{n+1} = \sum_{\ell,m=1,\dots,M} g^{m\ell}(n)\, VQ_\ell\, \Omega_n\, Q_m V^*\,, \tag{21}$$

$$V \;\text{ unitary on }\; \mathfrak{h}_A\,, \qquad g^{m\ell}(n) := \langle T^{(m)}\phi_{k_n}, T^{(\ell)}\phi_{k_n}\rangle \tag{22}$$

$G(n) := \big(g^{m\ell}(n)\big)$ is a non-negative matrix. Map $\Omega_n \mapsto \widehat{\Omega}_{n+1}$ given in (21) is *completely positive*. $\Rightarrow$ $\widehat{\Omega}_{n+1}$ is a density matrix. Spect. thm. $\Rightarrow$

$$\widehat{\Omega}_{n+1} = \sum_{j=1}^{L} p_j(n+1)\,\Pi_j(n+1), \quad p_1(n+1) > \cdots > p_L(n+1) > 0,$$

for some $L \leq M$, where the $\Pi_j(n+1)$ are orthogonal projections, and

$$\sum_{j=1}^{L} p_j(n+1)\,\mathrm{tr}\big(\Pi_j(n+1)\big) = 1\,.$$

According to the *Collapse Postulate*, **Axiom CP**, Nature chooses

The Weak-Coupling Regime

$$\Omega_{n+1} := \left[\mathrm{tr}\left(\Pi_{j*}\right)\right]^{-1} \Pi_{j*}(n+1)\,, \quad \text{for some} \;\; j_*\,, \tag{23}$$

as the state of the atom at time $n+1$, with

$$\text{Probablity} = p_{j*}(n+1)\mathrm{tr}\big(\Pi_{j*}(n+1)\big) \quad \text{(Born Rule)}$$

This proves $(\mathcal{I}_{n+1})$, thus completing the induction step.

The weak-coupling regime: $T^{(m)} = \mathbf{1} + \varepsilon\, \tau^{(m)}$, $\quad \|\tau^{(m)}\| \leq 1$, for some positive $\varepsilon \ll 1$. Then

$$g^{m\ell}(n) = 1 + \mathcal{O}(\varepsilon), \qquad \forall\; m, \ell.$$

Thus Eq. (21) implies

$$\widehat{\Omega}_{n+1} = V\,\Omega_n\,V^* + \mathcal{O}(\varepsilon) \;\Rightarrow\; \Omega_{n+1} = V\,\Omega_n\,V^* + \mathcal{O}(\varepsilon)\,, \tag{24}$$

with probability $1 - \mathcal{O}(\varepsilon)$, i.e., *time evolution of states* (in the Schrödinger picture) *is given, to a good approximation, by unitary conjugation!*

However, for *purely entropic reasons*, it happens with a frequency $\propto \varepsilon$ that $\mathrm{tr}(\Omega_{n+1} \cdot V\Omega_n V^*) \sim 0$. This is then perceived as an *"Event"* in the literal sense of the word!

The Strong-Coupling Regime

The strong-coupling regime: Characterized by

$$g^{m\ell}(n) = \langle T^{(m)}\,\phi_{k_n}, T^{(\ell)}\,\phi_{k_n}\rangle = \delta_{m\ell} + \mathcal{O}(\varepsilon), \quad 0 < \varepsilon \ll 1\,, \tag{25}$$

(at least for $k_n = 0$!) Then, for large enough times, n,

$$\widehat{\Omega}_{n+1} = \sum_{m=1}^{M} VQ_m\,\Omega_n\,Q_m V^* + \mathcal{O}(\varepsilon)\,, \tag{26}$$

hence $\Omega_{n+1} = VQ_k V^* + \mathcal{O}(\varepsilon)$, for some $k \in \{1, \dots, M\}$ (state collapse!). If $\Omega_n = V\,Q_\ell V^* + \mathcal{O}(\varepsilon)$ then the probability for Ω_{n+1} to be given by $\Omega_{n+1} = VQ_k V^* + \mathcal{O}(\varepsilon)$ is given by

$$P(k,\ell) := \mathrm{tr}\left(\left|Q_k V\,Q_\ell\right|^2\right). \tag{27}$$

Hence the evolution of states is well approximated by sample paths of a *classical Markov chain* with transition function $P(k,\ell)$!

Alternation Between Unitary Evolution and State Collapse

It can happen that the matrices $G(n) = \big(g^{m\ell}(n)\big)$ have the form $G(n) = G_0 + \mathcal{O}(\varepsilon)$, with

$$G_0 = \begin{pmatrix} 1 & \dots & 1 & 0 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 1 & \dots & 1 & 0 & \dots & 0 \\ 0 & \dots & 0 & 1 & \dots & 0 \\ \vdots & & \vdots & \vdots & & \vdots \\ 0 & \dots & 0 & 0 & \dots & 1 \end{pmatrix} \tag{28}$$

where the upper left block is a $K \times K$ matrix and the lower right block is the $(M-K) \times (M-K)$ identity matrix. Then unitary evolution prevails on the subspace $\mathfrak{h}_A^w$ of dimension K corresp. to the range of the proj. $\sum_{m=1}^{K} Q_m$, while on the complementary subspace $\mathfrak{h}_A^s = \mathfrak{h}_A \ominus \mathfrak{h}_A^w$ state collapse prevails. If the subsapces $\mathfrak{h}_A^w$ and $\mathfrak{h}_A^s$ are **not** invariant under V then there are q.m. **transitions** from one regime to the other regime in the course of the evolution of a state.

This leads to a succinct description of *measurements and observations* and of "*arrival times*" and *measurement times*.

Summary and Conclusions ...

- The *ETH-Approach to Quantum Mechanics* provides a logically coherent theory of *Potential and Actual Events*, of the recordings of the latter, and of measurements. It has resemblences with "Many Worlds," "GRW," ... ; yet, it supersedes these imprecise formalisms and describes but **One World**! The models in Sect. 4 provide a useful illustration of the *ETH*-Approach.
- As in the genesis of Special Relativity, *fields describing massless modes* (photons & gravitons), besides the even-dimensionality of space-time might play key roles in the genesis of a Quantum Theory that satisfies the spectrum condition ($H \geq 0$) and solves the "measurement problem." (Has not really been appreciated, so far!)
- *Actual Events* weave the fabric of space-time! ("Emergent gravity")
- Thanks to the *Principle of Diminishing Potentialities* (*PDP*) and the natural presence of an "arrow of time" in the *ETH*-Approach to Quantum Theory, the "Information –" and the "Unitarity Paradox" appear to dissolve. ...